Proof

P2 | Study notes, worked example and practice

What to understand

A proof is a chain of justified statements from assumptions to a conclusion. One example cannot prove a universal claim.

For proof by exhaustion, check every possible case in the stated finite set. A single valid counterexample disproves a universal claim.

Key formulae

$$2a + 2b = 2(a + b), \qquad a, b \in \mathbb{Z}$$

Worked example

2 is prime and even

The claim “all prime numbers are odd” is false: 2 is a counterexample.

Practice questions

Show your working. Try these without looking at the answers.

1. Disprove: every prime number is odd.

2. Prove by exhaustion that $n^2 - n$ is even for $n \in \{1, 2, 3, 4\}$.

3. Prove that the sum of two even integers is even.

Answers

Use these to check the method as well as the final answer.

1. 2 is prime and even.

2. The four values are 0, 2, 6 and 12; each is even.

3. Write the integers as 2a and 2b for integers a, b. Their sum is 2(a + b), which is even.