Algebra and functions

P2 | Study notes, worked example and practice

What to understand

The remainder theorem finds the remainder after dividing a polynomial by a linear expression.

The factor theorem says $x - a$ is a factor exactly when $f(a) = 0$. Divide to find the remaining factor.

Key formulae

$$\text{Remainder on division by } (x - a) = f(a)$$

$$(x - a) \text{ is a factor} \iff f(a) = 0$$

Worked example

$$f(x) = x^3 - 4x + 3, \qquad f(1) = 0$$

Therefore $x - 1$ is a factor. Division gives the quotient $x^2 + x - 3$.

Practice questions

Show your working. Try these without looking at the answers.

1. Find the remainder when $f(x) = x^3 - 2x + 3$ is divided by $x - 2$.

2. Show that $x - 1$ is a factor of $x^3 - 4x + 3$.

3. Divide $x^3 - 4x + 3$ by $x - 1$ and factorise the result fully.

Answers

Use these to check the method as well as the final answer.

1. $f(2) = 7$.

2. $f(1) = 1 - 4 + 3 = 0$.

3. Quotient $x^2 + x - 3$; hence $(x - 1)(x^2 + x - 3)$. The quadratic has real roots $(-1 \pm \sqrt{13})/2$.