Sequences and series

P2 | Study notes, worked example and practice

What to understand

Arithmetic sequences have a constant difference; geometric sequences have a constant ratio.

An infinite geometric sum exists only when the absolute value of the ratio is less than 1. The binomial formula below applies to non-negative integer powers.

Key formulae

$$u_n = a + (n-1)d, \qquad S_n = \frac{n}{2}[2a + (n-1)d]$$

$$u_n = ar^{n-1}, \qquad S_n = \frac{a(1-r^n)}{1-r}, \qquad S_\infty = \frac{a}{1-r} \quad (|r| < 1)$$

$$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k$$

Worked example

$$8 + 4 + 2 + \cdots = \frac{8}{1 - \frac{1}{2}} = 16$$

The first term is 8 and the common ratio is 1/2.

Practice questions

Show your working. Try these without looking at the answers.

1. Find the 10th term of the arithmetic sequence 3, 7, 11, … .

2. Find the sum of the first 5 terms of 2, 6, 18, … .

3. Find the sum to infinity of 8 + 4 + 2 + … .

4. Expand $(1 + 2x)^3$.

Answers

Use these to check the method as well as the final answer.

1. 39.

2. 242.

3. 16.

4. $1 + 6x + 12x^2 + 8x^3$.