Exponentials and logarithms

P2 | Study notes, worked example and practice

What to understand

A logarithm is an exponent: log base a of x is the power to which a must be raised to obtain x.

Logarithm laws require positive arguments. Check the domain when solving equations.

Key formulae

$$\log_a x = y \iff a^y = x$$

$$\log_a (xy) = \log_a x + \log_a y, \qquad \log_a (x^k) = k\log_a x$$

$$\log_a x = \frac{\log x}{\log a}$$

Worked example

$$\log_2 (x-1) = 3 \implies x - 1 = 2^3 \implies x = 9$$

The solution satisfies $x - 1 > 0$.

Practice questions

Show your working. Try these without looking at the answers.

1. Solve $2^x = 32$.

2. Simplify $\log_{10} 1000 - \log_{10} 10$.

3. Solve $\log_2(x - 1) = 3$.

4. Solve $3^x = 7$, giving an exact expression.

Answers

Use these to check the method as well as the final answer.

1. x = 5.

2. 2.

3. x = 9.

4. x = log 7 / log 3.