Trigonometry

P2 | Study notes, worked example and practice

What to understand

Use the fundamental trigonometric identities to simplify expressions and solve equations. Be alert to the specified interval and whether the question uses degrees or radians.

Find all solutions in the interval by using symmetry and periodicity. Check that each solution satisfies the original equation.

Key formulae

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\sin^2\theta + \cos^2\theta = 1$$

$$\sin(\theta + 360^\circ) = \sin\theta, \quad \cos(\theta + 360^\circ) = \cos\theta$$

$$\tan(\theta + 180^\circ) = \tan\theta$$

Worked example

$$2\sin\theta = 1 \quad \Longrightarrow \quad \sin\theta = \frac{1}{2}$$

For $0° \leq \theta < 360°$, sine is positive in the first and second quadrants.

$$\theta = 30^\circ,\ 150^\circ$$

Check both angles in the original equation.

Practice questions

Show your working. Try these without looking at the answers.

1. Find every solution.

$$\sin\theta = \frac{1}{2}, \qquad 0^\circ \leq \theta < 360^\circ$$

2. Find every solution.

$$\cos\theta = -\frac{1}{2}, \qquad 0^\circ \leq \theta < 360^\circ$$

3. Find every solution.

$$\tan\theta = 1, \qquad 0^\circ \leq \theta < 360^\circ$$

4. Find every solution.

$$2\sin^2\theta - 1 = 0, \qquad 0^\circ \leq \theta < 360^\circ$$

5. Simplify using a standard identity.

$$1 - \cos^2\theta$$

Answers

Use these to check the method as well as the final answer.

1. Sine is positive in quadrants I and II.

$$30^\circ, 150^\circ$$

2. Cosine is negative in quadrants II and III.

$$120^\circ, 240^\circ$$

3. Tangent has a period of 180°.

$$45^\circ, 225^\circ$$

4. First obtain $\sin\theta = \pm 1/\sqrt{2}$.

$$45^\circ, 135^\circ, 225^\circ, 315^\circ$$

5. Rearrange $\sin^2\theta + \cos^2\theta = 1$.

$$\sin^2\theta$$