Differentiation

P2 | Study notes, worked example and practice

What to understand

Use the first derivative to find stationary points, where the gradient is zero. Evaluate the original function to find each point’s y-coordinate.

Classify a stationary point using the second derivative or by checking how the gradient changes. Apply these ideas to maximum and minimum problems.

Key formulae

$$\frac{dy}{dx} = 0 \quad \text{at a stationary point}$$

$$\frac{d^2y}{dx^2} > 0 \Rightarrow \text{local minimum}$$

$$\frac{d^2y}{dx^2} < 0 \Rightarrow \text{local maximum}$$

Worked example

$$y = 12x - x^2, \qquad \frac{dy}{dx} = 12 - 2x$$

Setting the gradient to zero gives x = 6; then y = 36.

$$\frac{d^2y}{dx^2} = -2 < 0$$

The stationary point (6, 36) is a maximum.

Practice questions

Show your working. Try these without looking at the answers.

1. Find and classify the stationary point.

$$y = x^2 - 6x + 5$$

2. Find the maximum value and where it occurs.

$$y = 12x - x^2$$

3. Find both stationary points and classify them.

$$y = x^3 - 3x$$

4. State where the function is increasing.

$$y = x^2 - 4x$$

Answers

Use these to check the method as well as the final answer.

1. The derivative is $2x - 6$ and the second derivative is 2.

$$(3, -4) \quad \text{minimum}$$

2. The second derivative is -2.

$$(6, 36) \quad \text{maximum value} = 36$$

3. The derivative is $3x^2 - 3$ and the second derivative is $6x$.

$$(-1, 2)\ \text{maximum}; \qquad (1, -2)\ \text{minimum}$$

4. The derivative $2x - 4$ is positive for $x > 2$.

$$x > 2$$