Integration

P2 | Study notes, worked example and practice

What to understand

A definite integral gives signed area. The area below the x-axis is counted as negative by the integral; geometric area is positive.

When finding area between two curves, integrate the upper function minus the lower function over the stated limits.

The trapezium rule gives an estimate from equally spaced ordinates.

Key formulae

$$\int_a^b f(x)\,dx = F(b) - F(a), \qquad F'(x) = f(x)$$

$$\text{Area} = \int_a^b [\text{upper curve} - \text{lower curve}]\,dx$$

$$\text{Trapezium rule}: \quad T = \tfrac{h}{2}[y_0 + 2y_1 + \cdots + 2y_{n-1} + y_n]$$

Worked example

$$\int_0^3 2x\,dx = [x^2]_0^3 = 9$$

Evaluate the antiderivative at the upper limit and subtract its value at the lower limit.

Practice questions

Show your working. Try these without looking at the answers.

1. Evaluate the integral.

$$\int_0^2 3x^2\,dx$$

2. Find the area under the curve and above the x-axis.

$$y = x^2, \quad x = 0, \quad x = 3$$

3. Find the area between the curves.

$$y = 2x, \quad y = x^2, \quad 0 \leq x \leq 2$$

4. Use the trapezium rule with two strips to estimate the integral from 0 to 2.

$$y = x^2, \quad x = 0, 1, 2$$

Answers

Use these to check the method as well as the final answer.

1. The antiderivative is x^3.

$$[x^3]_0^2 = 8$$

2. The curve is on or above the x-axis on this interval.

$$\left[\frac{x^3}{3}\right]_0^3 = 9$$

3. Here $2x$ is the upper curve.

$$\int_0^2 (2x - x^2)\,dx = \frac{4}{3}$$

4. The width is 1. The exact integral is 8/3, so this estimate is an overestimate.

$$T = \frac{1}{2}(0 + 2 \times 1 + 4) = 3$$